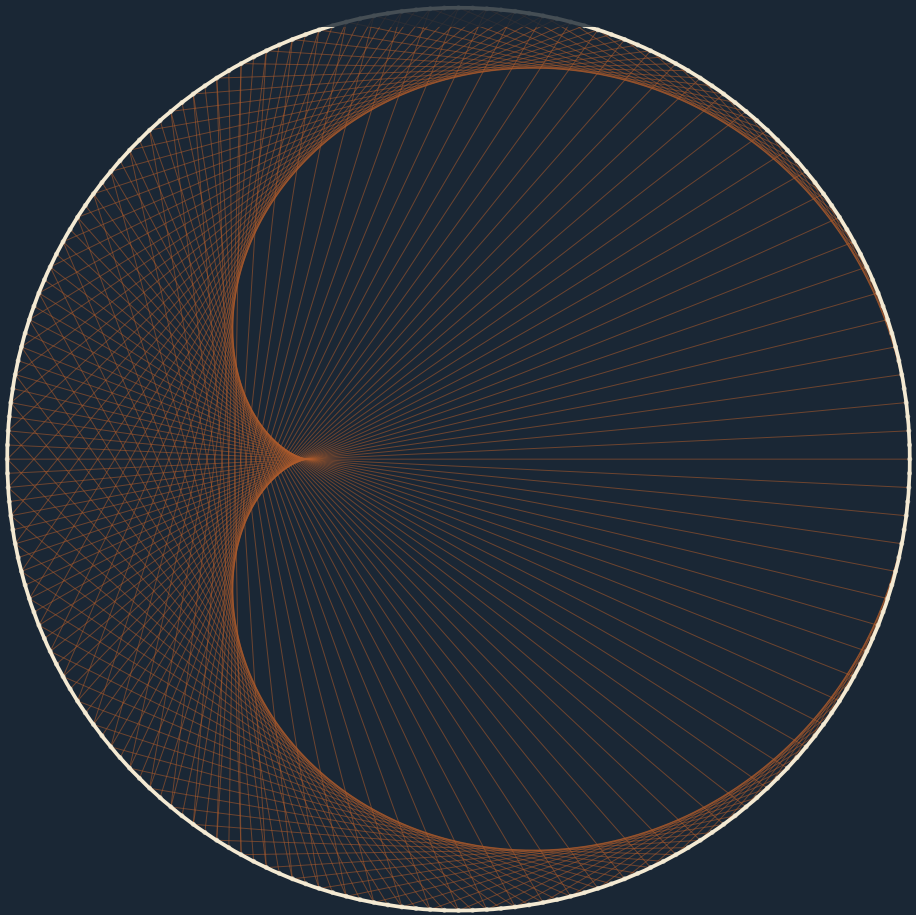


CURATED MATHEMATICS

Number Puzzles



Fifty classic puzzles, with worked solutions



VAMSHI JANDHYALA · LONDON

- Leave the 31 litre cask: 72 litres are sold, 24 of brandy and 48 of wine. No selection of 8, 13, 15, 17, 19 adds up to 24, so this fails as well.
- Leave the 19 litre cask: 84 litres are sold, 28 of brandy and 56 of wine. This works: the brandy is $13 + 15 = 28$ litres, and the wine is $8 + 17 + 31 = 56$ litres.

So the wine sells at 50 pence a litre and the brandy at £1 a litre. The wine buyer pays $56 \times 0.5 = 28$ pounds and the brandy buyer $28 \times 1 = 28$ pounds, exactly as required. The unsold cask is the 19 litre one, and as brandy it is worth 19 pounds, that is £19.

PUZZLE 3

The Missing Term



A student was copying out a sequence for his homework when the telephone rang. While he was gone, his younger brother rubbed out one of the terms. Here is what was left:

10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 24, ____, 100, 121, 10000.

What was the number that was rubbed out?

Solution

Every term is the number sixteen, written in a different base, with the bases counting down from sixteen to two.

In base sixteen, sixteen is written 10; in base fifteen, 11; in base fourteen, 12; and so on down to base ten, where it is the familiar 16, then base nine (17), and base eight (20, since $16 = 2 \times 8$), base seven (22), base six (24). The next base down is five, and $16 = 3 \times 5 + 1$, so there sixteen is written 31. The list

then finishes with base four (100, since $16 = 4^2$), base three (121, since $16 = 9 + 6 + 1$) and base two (10000).

The missing term is 31, which is sixteen written in base five.

PUZZLE 4

The Club of Six



Six friends, the men Tom, Dick and Harry and the women Anna, Cathy and Lucy, make up three married couples. Which man is married to which woman is exactly what we are asked to find.

One day they each went to market and bought some sheep. By chance, each of them bought as many sheep as the number of pounds they paid for a single one, so a person who bought n sheep paid n pounds apiece and spent n^2 pounds in all.

Comparing receipts afterwards, they noticed two things. Each husband had spent exactly £63 more than his own wife. And Tom had bought 23 more sheep than Cathy, while Dick had bought 11 more than Lucy.

Who is married to whom?

Solution

If a husband bought h sheep and his wife w , he spent h^2 pounds and she spent w^2 , and we are told $h^2 - w^2 = 63$. Written as $(h - w)(h + w) = 63$, and since 63 is odd both factors must be odd. The three ways to split 63 into two odd factors,

$$(h - w, h + w) = (1, 63), \quad (3, 21), \quad (7, 9),$$

give $(h, w) = (32, 31), (12, 9)$ and $(8, 1)$. So the husbands bought 32, 12 and 8 sheep, and the wives bought 31, 9 and 1.

Now use the clues. Tom bought 23 more than Cathy. Among the husbands' counts 32, 12, 8 and the wives' counts 31, 9, 1, the only difference equal to 23 is $32 - 9$, so Tom bought 32 and Cathy bought 9. Dick bought 11 more than Lucy; of the men left, 12 and 8, and the women left, 31 and 1, the only difference equal to 11 is $12 - 1$, so Dick bought 12 and Lucy bought 1. That leaves Harry with 8 and Anna with 31.

Finally, pair each husband with his wife using the counts found at the start: the man who bought 32 is married to the wife who bought 31, the one who bought 12 to the wife who bought 9, and the one who bought 8 to the wife who bought 1. Therefore

Tom is married to Anna, Dick to Cathy, Harry to Lucy.

The pleasing twist is that neither woman named in the clues is the wife of the man she is compared with.

PUZZLE 5

One Hundred, and One



1. Take the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 in that order. Placing only plus and minus signs between them, and running neighbours together into larger numbers where you wish, make the result equal to 100.
2. Using all ten digits 0 through 9, each exactly once, write an expression equal to 1.

Solution

Part 1. A tidy answer using only three signs is

$$123 - 45 - 67 + 89 = 100.$$

CURATED MATHEMATICS

Number Puzzles



Fifty classic puzzles, with worked solutions

Vamshi Jandhyala

LONDON

Number Puzzles

First published in Great Britain in 2026 by Vamshi Jandhyala, London.

Copyright © Vamshi Jandhyala 2026.

The moral right of the author has been asserted.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means without the prior written permission of the author, nor be otherwise circulated in any form of binding or cover other than that in which it is published.

Fifty classical number puzzles, recast in contemporary language and given fresh, fully worked solutions.

A catalogue record for this book is available from the British Library.

ISBN 978-1-8000-0011-7 (paperback)

Typeset in TeX Gyre Pagella by the author.



A Note on These Puzzles



This is a small collection of classic number puzzles, the kind that have circulated for a century or more in magazines, common rooms and the backs of notebooks. Each has been chosen for one reason: that it hides a genuinely pretty piece of reasoning, and rewards you the moment you see it.

Every puzzle is its own short chapter. The statement comes first, set in plain modern language, and the solution follows at once, worked through in full rather than merely announced. The aim throughout is the shortest honest argument, the one that makes you nod rather than the one that grinds to the answer. Where a puzzle turns on a single idea, casting out nines, a difference of two squares, counting in threes, the idea is named and explained, so that the same trick is yours to use again.

Nothing here asks for mathematics beyond school. A reader comfortable with ordinary arithmetic, a little algebra and the patience to follow a clear chain of reasoning will be able to read every solution to the end. A few puzzles brush against deep results, and where a fact has been settled by mathematics heavier than these pages can carry, this is said plainly and the result is taken on trust rather than dressed up as obvious.

The puzzles can be read in any order. Cover the solution, attempt the puzzle, and consult the working only when you are ready. Half the pleasure is in the attempt, and the other half in seeing how little, in the end, the answer really needed.

Contents



1	<i>The Suppressed Digit</i>	1
2	<i>Brandy and Wine</i>	2
3	<i>The Missing Term</i>	3
4	<i>The Club of Six</i>	4
5	<i>One Hundred, and One</i>	5
6	<i>A Stroke of Genius</i>	6
7	<i>Two Games of Chance</i>	7
8	<i>Dots in Triangles and Pyramids</i>	8
9	<i>Jack and Jill</i>	9
10	<i>Square Palindromes</i>	10
11	<i>Four Consecutive Numbers</i>	11
12	<i>Divisibility by Seventy-Three</i>	12
13	<i>Pick Five, Always Sixty-Five</i>	13
14	<i>Cancelling, and a Cube Trick</i>	14
15	<i>A Pandigital Divisible by Eleven</i>	16
16	<i>Casting Out in Threes</i>	17
17	<i>A Prime Number of Rails</i>	18
18	<i>Nine Digits, the Unit Fractions</i>	19
19	<i>A Father's Fair Division</i>	19
20	<i>A Will of Diamonds</i>	21

Contents

21	<i>Four Weights for Forty</i>	22
22	<i>One Light Column</i>	22
23	<i>The Ages of Three Children</i>	23
24	<i>Numbers Made of Their Own Factorials</i>	24
25	<i>Ten in a Row, None Prime</i>	25
26	<i>A Power of Every Kind</i>	26
27	<i>His Age in a Square Year</i>	27
28	<i>One Before, One After</i>	28
29	<i>A Number That Counts Itself</i>	28
30	<i>The Coconuts and the Monkey</i>	29
31	<i>The House in the Middle</i>	30
32	<i>Numbers That Reappear in Their Squares</i>	31
33	<i>A Cube and a Square</i>	32
34	<i>Threading Thirty-One and Seventy-Three</i>	33
35	<i>Eight Queens</i>	34
36	<i>Weighed Two at a Time</i>	35
37	<i>Squares of All the Digits</i>	36
38	<i>Every Digit, Every Divisor</i>	37
39	<i>Adding a Million Numbers at Once</i>	37
40	<i>Matched at Every Power</i>	38
41	<i>ADE plus ODO</i>	39
42	<i>What the Post Office Cannot Make</i>	40
43	<i>The Greatest with Three Digits</i>	41
44	<i>The Only Magic Hexagon</i>	42
45	<i>Three Urns</i>	43

Contents

<i>46 One Hundred from Nine Digits</i>	<i>44</i>
<i>47 A Thousand Pounds in Bags</i>	<i>44</i>
<i>48 An Ancient Magic Square</i>	<i>45</i>
<i>49 The Counting Competition</i>	<i>46</i>
<i>50 Bananas and Pears</i>	<i>47</i>

The Suppressed Digit



Here is a trick to play on a friend. Ask them to do the following out of your sight.

1. Write down any whole number with more than five digits.
2. Add up its digits, and subtract that total from the number itself.
3. From the result, cross out any one digit that is not a zero.
4. Read out the digits that remain, in any order they please.

You then name the digit they crossed out. If they read out 2, 2, 3, 5, 9, you would tell them at once that the suppressed digit was a 6. How is it done?

Solution

The trick rests on a single fact about the number nine: a whole number and the sum of its digits leave the same remainder on division by nine. This holds because 10, 100, 1000 and so on are each one more than a multiple of nine, so every digit contributes only itself to the remainder.

It follows that subtracting the digit sum from the number always leaves an exact multiple of nine. And the digits of a multiple of nine themselves add up to a multiple of nine.

So the digits of the result, all of them together, add up to a multiple of nine. The ones read out to you sum to $2 + 2 + 3 + 5 + 9 = 21$. The next multiple of nine above 21 is 27, so the suppressed digit must be $27 - 21 = 6$.

This also explains the rule against crossing out a zero. If the digits you hear already add up to a multiple of nine, the hid-

den digit is a 9, not a 0, since a suppressed 0 and a suppressed 9 could not be told apart. Barring zero removes the ambiguity.

PUZZLE 2

Brandy and Wine



A merchant has six casks, holding 8, 13, 15, 17, 19 and 31 litres. He stocks only two drinks, brandy and wine, and his brandy costs exactly twice as much per litre as his wine.

Two customers arrive. The first buys only brandy and spends exactly £28. The second buys only wine and also spends exactly £28. No cask is ever broken into: each is sold whole or not at all. When the customers leave, a single cask remains unsold.

Taking that last cask to hold brandy, what is it worth?

Solution

Both customers spend the same £28, but brandy costs twice as much per litre as wine, so the wine buyer must have carried off exactly twice the volume of the brandy buyer.

The five casks that were sold therefore split into a brandy share and a wine share twice as large. Together they come to three times the brandy volume, so the volume sold is a multiple of three.

All six casks hold $8 + 13 + 15 + 17 + 19 + 31 = 103$ litres. For the sold volume to be a multiple of three, the unsold cask must leave a multiple of three behind. Looking at remainders on division by three, only the 13, 19 and 31 litre casks qualify.

- Leave the 13 litre cask: 90 litres are sold, 30 of brandy and 60 of wine. No selection of 8, 15, 17, 19, 31 adds up to 30, so this fails.