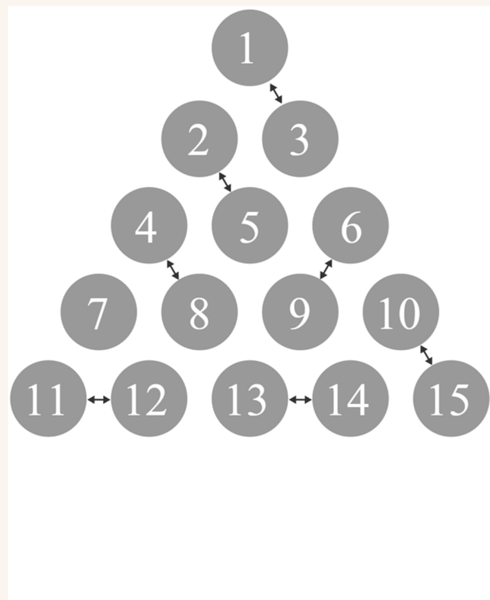
Vamshi Jandhyala

Dancer Pairs

2024

Fifteen dancers are standing in an equilateral triangle formation, with every dancer standing 1 unit apart from her nearest neighbours. Each dancer chooses another who is 1 unit away to pair up with, and all but 1 dancer ends up as part of a pair. An example of one such arrangement is shown below. How many different sets of 7 pairs are possible?



Solution

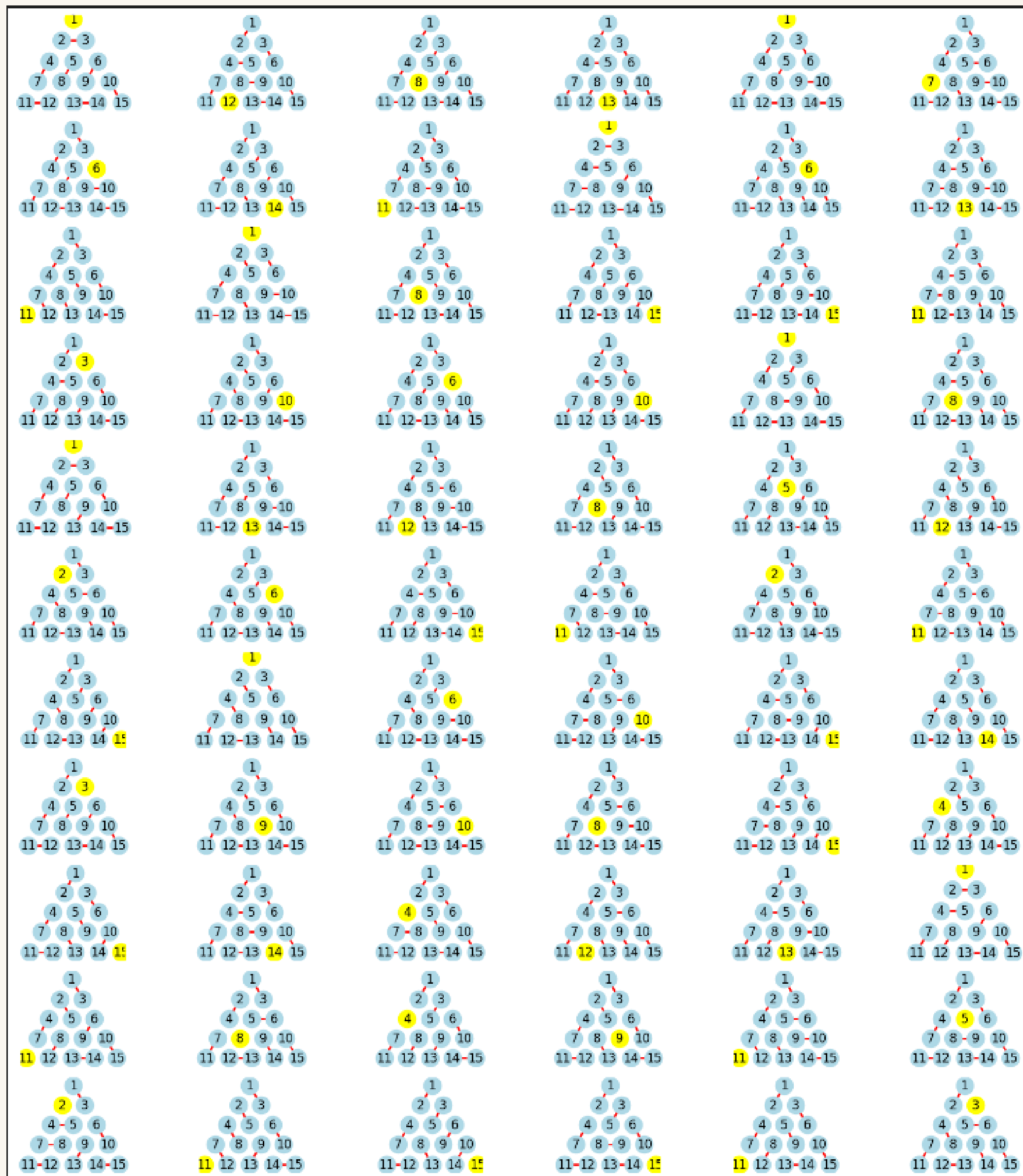
We use a backtracking algorithm in `generate_all_pairings` to generate all possible combinations. The `backtrack` function works as follows:

- It takes the current index, the current pairing, and the set of used nodes.
- If we have 7 pairs, add the current pairing to our list of all pairings.
- For each node from the current index onwards:
  - If the node is already used, skip it.

  - For each neighbour of the node: if the neighbour is not used and has a higher index (to avoid duplicates), create a new pairing with this pair, then recursively call `backtrack` with the updated pairing and used nodes.

- We use `frozensets` to represent pairs and combinations of pairs, allowing duplicates to be eliminated easily.

The code below shows that there are **240** ways of pairing the dancers. A few sample pairings:



Python code

```
import networkx as nx
from itertools import combinations
import matplotlib.pyplot as plt
```

```python
import math

def create_dancer_graph():
    G = nx.Graph()
    edges = [
        (1, 2), (1, 3),
        (2, 3), (2, 4), (2, 5),
        (3, 5), (3, 6),
        (4, 5), (4, 7), (4, 8),
        (5, 6), (5, 9), (5, 8),
        (6, 9), (6, 10),
        (7, 8), (7, 11), (7, 12),
        (8, 9), (8, 13), (8, 12),
        (9, 10), (9, 13), (9, 14),
        (10, 14), (10, 15),
        (11, 12), (12, 13), (13, 14), (14, 15),
    ]
    G.add_edges_from(edges)
    return G

def generate_all_pairings(G):
    all_pairings = []
    nodes = list(G.nodes())

    def backtrack(index, current_pairing, used_nodes):
        if len(current_pairing) == 7:
            all_pairings.append(frozenset(map(frozenset, current_pairing)))
            return
        for i in range(index, len(nodes)):
            node = nodes[i]
            if node in used_nodes:
                continue
            for neighbor in G.neighbors(node):
                if neighbor not in used_nodes and neighbor > node:
                    new_pairing = current_pairing + [(node, neighbor)]
                    new_used = used_nodes | {node, neighbor}
                    backtrack(i + 1, new_pairing, new_used)

    backtrack(0, [], set())
    return set(all_pairings)

G = create_dancer_graph()
all_combinations = generate_all_pairings(G)
print(f"Total number of combinations: {len(all_combinations)}")
```