Vamshi Jandhyala

Can You Spell Like a Queen Bee?

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A puzzle from Fiddler on the Proof, about how much a rounded percentage gives away.

Problem

In the New York Times' Spelling Bee, finding every word earns the rank of Queen Bee. That maximum score is never displayed, but the cutoff for Genius is, and it is 70 per cent of the maximum, rounded to the nearest whole number. Dividing the Genius cutoff by 0.7 therefore approximates the maximum, but does not always pin it down, since several maxima can round to the same cutoff.

For a very large, randomly chosen maximum, what is the probability that it can be determined exactly from its Genius cutoff?

Solution

Write M for the maximum and $G(M) = \mathrm{round}(0.7M)$ for the Genius cutoff. We want the density of those M that no other maximum shares a cutoff with.

The useful thing to watch is not G itself but how it changes. Step M up by one and $0.7M$ rises by 0.7, so G either stays put or climbs by one, and never does anything else. Two maxima collide exactly when G stays put, and because $0.7 > \frac{1}{2}$ it can never stay put twice running, so collisions come in pairs and never in threes.

That is the whole argument, and it can be finished by bookkeeping. Over a long run of M, the cutoff G has to climb at an average rate of 0.7 per step, so a fraction 0.7 of the steps climb and a fraction 0.3 stand still. Each of those stationary steps spoils exactly two values of M, the one on each side of it, and no value is spoiled twice because no two stationary steps are adjacent. So the ambiguous maxima have density 2×0.3, and

$$\boxed{\Pr(M \text{ is determined}) \;=\; 1 - 2(1 - 0.7) \;=\; \frac{2}{5}.}$$

It is worth seeing the cycle itself, since $0.7 = 7/10$ means everything repeats every ten. Over one period, ten maxima are squeezed onto seven cutoffs:

M	0	1	2	3	4	5	6	7	8	9
G	0	1	1	2	3	4	4	5	6	6
determined?	✓			✓	✓			✓		

Seven cutoffs for ten maxima means three of the cutoffs are used twice, which spoils six maxima and leaves four. Four in ten is $2/5$, as the counting said.

The argument never used the digit 7, only that 0.7 lies between a half and one, so the general statement is that a cutoff at a fraction p of the maximum determines it with probability $2p-1$. At $p = \frac{1}{2}$ nothing is ever determined, which is right, since then every cutoff serves two maxima. The answer improves linearly as the cutoff climbs towards the maximum itself.

Extra Credit

Set Genius aside. The other ranks are Amazing at 50 per cent, Great at 40, Nice at 25, Solid at 15, Good at 8, Moving Up at 5, and Good Start at 2, each rounded to the nearest whole number. For a very large, randomly chosen maximum, what is the probability that it can be determined exactly from these seven cutoffs together?

Two of the seven do no separating at all

Start with Amazing, the sharpest of them. Rounding half upwards, $\mathrm{round}(M/2)$ is k for $M = 2k$ and also k for $M = 2k-1$, since $k-\frac{1}{2}$ rounds up. So Amazing reads the same for $2k-1$ and $2k$ and differently for everything else. On its own it sorts the maxima into the pairs

$$\{1,2\},\ \{3,4\},\ \{5,6\},\ \ldots,\ \{2k-1,\ 2k\},\ \ldots$$

and that is as far as it goes. Every remaining question is about one pair: a maximum is determined precisely when one of the other six cutoffs tells it apart from its partner.

Good Start turns out to be no help either, and for the same structural reason. Its cutoff $\mathrm{round}(M/50)$ steps up as M passes $50n + 25$, an odd number, so the step falls between an even M and the odd one after it. That is the gap *between* two pairs, never the gap inside one. Amazing locates the pair and Good Start agrees with it; neither can look inside.

So five cutoffs are left to do the work: 5, 8, 15, 25 and 40 per cent.

Counting what is left

A pair $\{2k-1, 2k\}$ survives as an ambiguity exactly when all five of those cutoffs read the same on both halves. Cutoff p separates them when a rounding boundary falls between $p(2k-1)/100$ and $p(2k)/100$, an interval of width $p/100$, so whether it does depends only on the residue of M. The percentages 5, 15, 25 and 40 all have denominator dividing 20 once written over 100, and 8 per cent turns out never to be the deciding vote, so the whole pattern repeats every 20 rather than every 100.

That makes the count small enough to write down. Of every twenty consecutive maxima, exactly six are ambiguous,

$$M \equiv 0,\ 7,\ 8,\ 15,\ 16,\ 19 \pmod{20},$$

which is three of the ten pairs: $\{19,0\}$, $\{7,8\}$ and $\{15,16\}$. Hence

$$\boxed{\Pr(M \text{ is determined}) \;=\; 1-\tfrac{6}{20} \;=\; \tfrac{7}{10}.}$$

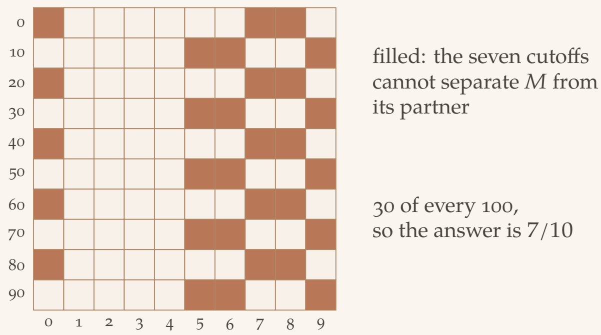

Figure 1: *The residues of M modulo 100, tens down the side and units along the bottom. A filled cell is a maximum the seven cutoffs cannot pin down, because its partner under Amazing reads identically on all of them. They come in adjacent pairs, thirty cells in all.*

Seven cutoffs beat one, but not by as much as one might hope: 7/10 against 2/5. The reason is that they are far from independent. All seven are rounded percentages of the same number, so they tend to break at the same places, and five of them are doing all the work. Adding ranks has diminishing returns quickly.

A caveat about halves

This answer depends on how ties are rounded, and the dependence is not cosmetic. Amazing at 50 per cent lands exactly on a half for every odd maximum, which is as common a tie as one can arrange. Rounding halves upwards, the standard reading of *rounded to the nearest whole number*, gives 7/10. Rounding halves to the nearest even number instead gives 9/20, a materially different answer. The main puzzle is not sensitive in this way and gives 2/5 either way, because at 70 per cent the ties are rarer and fall where they do no damage.

The computation

Both parts are finite counts once the period is known, so the check enumerates the cutoffs directly and looks for collisions, using exact rational arithmetic so that no tie is decided by floating point.

```
import math
from fractions import Fraction
from collections import Counter

def rnd(x):                              # nearest whole number, halves up
    return math.floor(x + Fraction(1, 2))

def cuts(M, pcts):
    return tuple(rnd(Fraction(p*M, 100)) for p in pcts)

def density(pcts, lo=100_000, span=20_000):
    T = {M: cuts(M, pcts) for M in range(lo-300, lo+span+300)}
    c = Counter(T.values())
    return Fraction(sum(c[T[M]] == 1 for M in range(lo, lo+span)), span)
```

```
print(density([70]))                    # 2/5
print(density([2, 5, 8, 15, 25, 40, 50])) # 7/10
```

Twenty thousand consecutive maxima, well away from zero, give exactly $2/5$ and $7/10$. The same enumeration confirms the structural claims: every collision group has size two and never three, every colliding pair is of the form $(2k-1, 2k)$, and across four thousand pairs the 2 and 50 per cent cutoffs separate none of them, while 25 per cent separates half, 40 per cent two fifths, 15 per cent a fifth, 5 per cent a tenth and 8 per cent two twenty-fifths.