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Can You Power up the Hill?

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A Tour de Fiddler cycling-physics puzzle from The Fiddler.[1]

Problem

This year's Tour de Fiddler sets two riders against a hill. A cyclist's speed along the ground is modelled by

$$v = \frac{P}{m \sin\theta + 10},$$

where P is pedalling power, m is mass, and θ is the angle of the incline. Riders mark a road by its gradient g, the slope written as a percentage, so a 45° wall has gradient 1; that is, $g = \tan\theta$. A climber has power 300 and mass 60; a sprinter has power 325 and mass 80. At what gradient do the two ride at the same speed?

For extra credit, the pair race up a perfectly sinusoidal hill, from a base where the gradient is 0 to a peak where it is again 0. What must the hill's maximum gradient be for them to reach the top at the same time? The speed above is measured along the ground, so on a slope the same speed covers less horizontal ground per unit time.

Solution

Both riders obey the same law, $v = P/(m \sin\theta + 10)$, so matching their speeds on a slope of angle θ means

$$\frac{300}{60 \sin\theta + 10} = \frac{325}{80 \sin\theta + 10}.$$

This is gentler than it looks. Each speed is a ratio, so clearing the two denominators turns the equation into a plain linear one in $\sin\theta$, with the awkward squared and trigonometric pieces never appearing. Cross-multiplying,

$$300\,(80 \sin\theta + 10) = 325\,(60 \sin\theta + 10),$$

and expanding gives $24000 \sin\theta + 3000 = 19500 \sin\theta + 3250$, so $4500 \sin\theta = 250$ and

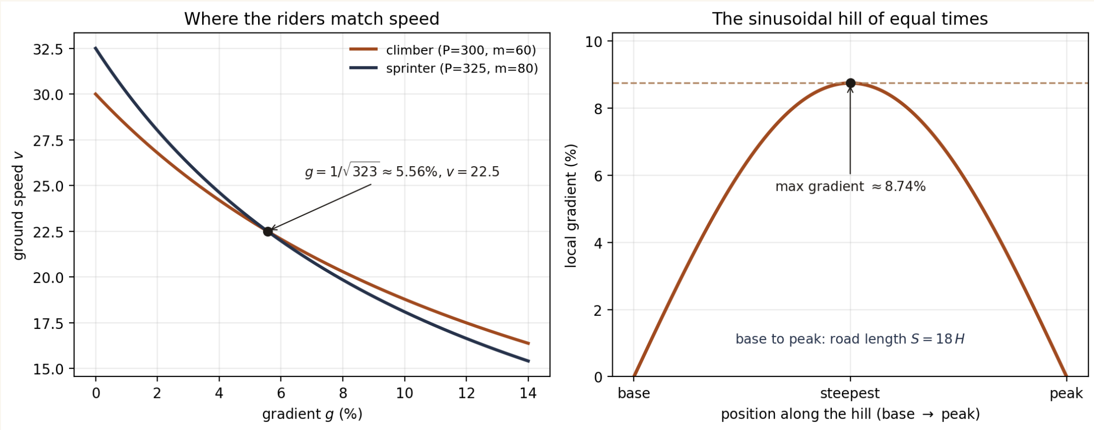
$$\sin\theta = \frac{250}{4500} = \frac{1}{18}.$$

[1] Zach Wissner-Gross, *The Fiddler*, 10 July 2026, thefiddler.substack.com.

At this incline both riders travel at $300/(60 \cdot \frac{1}{18} + 10) = 22.5$, the same speed. The gradient is the slope $g = \tan\theta$, and with $\sin\theta = \frac{1}{18}$ the cosine is $\cos\theta = \sqrt{1 - \frac{1}{324}} = \frac{\sqrt{323}}{18}$, so

$$g = \tan\theta = \frac{1/18}{\sqrt{323}/18} = \boxed{\frac{1}{\sqrt{323}} \approx 0.0556 \ \ (5.56\%).}$$

The riders trade advantages with the slope. On the flat the sprinter's greater power wins outright, 32.5 against the climber's 30; on a steep wall the climber's lighter frame tells; and the crossover sits at a gentle 5.56%.



Left: each rider's ground speed against gradient; the curves cross at $g = 1/\sqrt{323} \approx 5.56\%$, *both at speed* 22.5. *Right: the gradient along the sinusoidal hill, whose road length must be* 18 *times the rise, forcing a maximum gradient near* 8.74%.

Extra credit

The hill looks forbidding, with the speed changing continuously all the way up, but one observation flattens it. The time to cover a short stretch of road of length ds at local angle θ is $dt = ds/v = (m\sin\theta + 10)\,ds/P$, so the total climbing time is

$$T = \frac{1}{P}\left(m\int \sin\theta\,ds + 10\int ds\right).$$

Now $\sin\theta$ is exactly the vertical rise per unit of road travelled, since a step ds along ground tilted at θ lifts the rider by $dy = \sin\theta\,ds$. So the first integral is the total height climbed and the second is the total length of road,

$$T = \frac{mH + 10S}{P}, \qquad H = \text{total rise}, \quad S = \text{total road length}.$$

Every twist of the hill has cancelled out. Only the height gained and the length of tarmac survive, and both riders face the same H and S. Equal times therefore ask only that

$$\frac{60H + 10S}{300} = \frac{80H + 10S}{325},$$

which reduces, exactly as the main puzzle did, to $250S = 4500H$, so $S = 18H$: the road must be eighteen times as long as the hill is tall.

The word "sinusoidal" turns that length into a gradient. Take the profile $y(x) = \frac{H}{2}(1 - \cos(\pi x/W))$ across a horizontal span W: it climbs from base to peak with zero slope at each end, and its steepest point, the middle, has gradient $G = H\pi/(2W)$. The road length is the arc length,

$$S = \int_0^W \sqrt{1 + y'(x)^2}\, dx = \frac{H}{G}\sqrt{1 + G^2}\, E\left(\frac{G}{\sqrt{1 + G^2}}\right),$$

with E the complete elliptic integral of the second kind. Imposing $S = 18H$ leaves a single equation for the maximum gradient,

$$\sqrt{1 + G^2}\, E\left(\frac{G}{\sqrt{1 + G^2}}\right) = 18\,G,$$

whose root is

$$\boxed{G \approx 0.0874 \ \ (8.74\%).}$$

A hill this shallow is barely longer than its horizontal run, so to a first approximation $S \approx W$ and $G \approx \pi/36 \approx 8.73\%$; the small extra length the curve carries nudges the true maximum up to 8.74%.

Python code

The identity $T = (mH + 10S)/P$ is a shortcut, so the honest check throws it away and rides the actual hill. Cut the sinusoidal road into short ground segments; on each read the local slope, turn it into an angle, take the speed from the puzzle's formula; then sum ds/v. For the main puzzle, find the gradient where the two speed curves cross; for the extra credit, the maximum gradient where the two total times agree.

```
import numpy as np
from scipy.optimize import brentq
from scipy.special import ellipe

def speed(P, m, theta):      # ground-speed model from the puzzle
   return P / (m*np.sin(theta) + 10.0)

# MAIN: gradient g = tan(theta) where climber and sprinter match speed.
th = brentq(lambda t: speed(300,60,t) - speed(325,80,t), 1e-9, np.pi/2-1e-9)
print(f"MAIN: sin(theta) = {np.sin(th):.6f} (1/18 = {1/18:.6f})")
print(f"    speeds {speed(300,60,th):.4f} = {speed(325,80,th):.4f}")
print(f"    g = tan(theta) = {np.tan(th):.6f} = {100*np.tan(th):.3f}%")

# EC: integrate the REAL climb up a sinusoidal hill, dt = ds / v along the
# ground, and find the max gradient G making the two total times equal.
def climb_time(P, m, G, H=1.0, N=400_000):
   W = H*np.pi/(2*G)                     # max gradient G = H*pi/(2W)
   x = np.linspace(0.0, W, N+1)
   y = (H/2)*(1 - np.cos(np.pi*x/W))     # sinusoidal profile, rise H
   ds = np.hypot(np.diff(x), np.diff(y)) # ground-path segments
   theta = np.arctan2(np.diff(y), np.diff(x)) # local incline angle
   return np.sum(ds / speed(P, m, theta)) # sum of ds / v

G = brentq(lambda g: climb_time(300,60,g) - climb_time(325,80,g), 0.05, 0.15)
print(f"EC: max gradient G = {G:.6f} = {100*G:.3f}%")
print(f"    equal times: {climb_time(300,60,G):.5f} vs {climb_time(325,80,G):.5f}")
```

```
# cross-check the shape-independent identity S = 18 H via the closed form
# S/H = sqrt(1+G^2) E(k)/G, k^2 = G^2/(1+G^2):
G2 = brentq(lambda g: np.sqrt(1+g*g)*ellipe(g*g/(1+g*g))/g - 18.0, 0.05, 0.15)
print(f"    closed-form S=18H gives G = {G2:.6f} (matches)")
# MAIN: sin(theta) = 0.055556 (1/18 = 0.055556)
#       speeds 22.5000 = 22.5000
#       g = tan(theta) = 0.055641 = 5.564%
# EC:  max gradient G = 0.087433 = 8.743%
#       equal times: 0.80000 vs 0.80000
#       closed-form S=18H gives G = 0.087433 (matches)
```