

Can You Catch the Longest Wave?

 June 2026

A geometric-optimisation puzzle from *The Fiddler*.¹

Problem

Semicircle Island is shaped like a perfect semicircle, a semidisk of radius 1 mile, with its straight shoreline facing the open sea. A single wave, a thin straight wall of water that never changes speed or direction, makes first contact with the island at exactly 10 a.m. and last contact at 10:10 a.m. A surfer rides the wave where it meets the land. At 10:05 a.m., the midpoint of the wave's ten-minute crossing, what is the longest possible stretch of island directly under the wave?

For extra credit: if the wave's direction is now uniformly random, with every heading equally likely, what is the average length of land under the wave at the midpoint of its crossing?

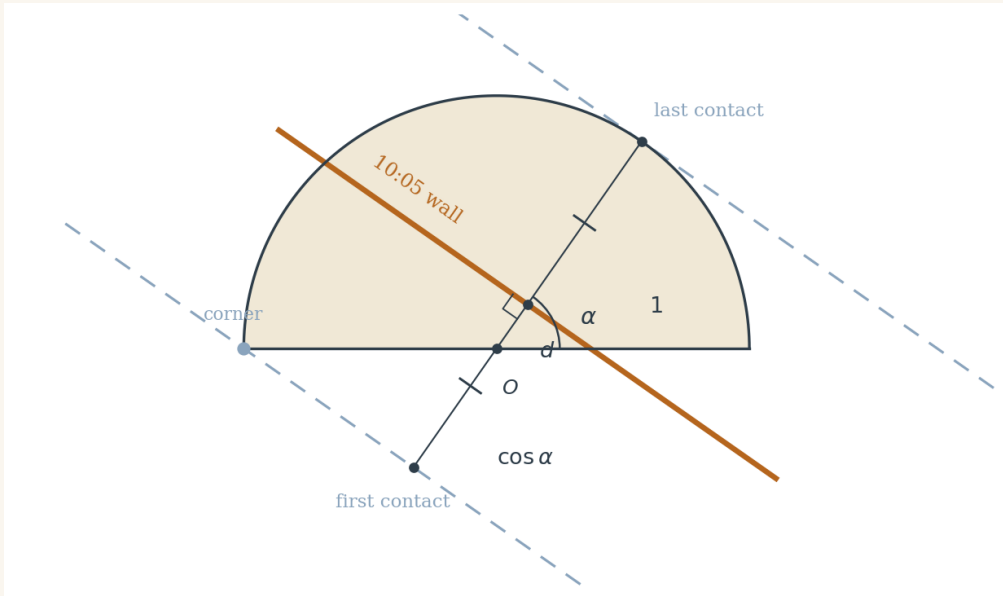
Solution

Put the island as the top half of a unit circle, with its centre O in the middle of the straight shore, and let the wave roll in off the sea at angle α to the shore.

Because the wave moves at a steady speed, equal times are equal distances travelled. It first touches the island at 10:00 and clears it at 10:10, so at 10:05, halfway through in time, it has gone exactly halfway in distance. The 10:05 wall therefore sits midway between the line of first contact and the line of last contact, and those two lines are easy to pin down. Both rest against the island and are square-on to the wave's travel, so the three lines are parallel. Measuring distance from O along the direction of travel, last contact grazes the round arc, a distance 1 from O , while first contact rests on the near corner $(-1, 0)$, a distance $\cos \alpha$ from O on the opposite side. The 10:05 wall is halfway between, so its distance from O is the average of the two,

$$d = \frac{1 - \cos \alpha}{2}.$$

¹Zach Wissner-Gross, *The Fiddler*, 12 June 2026, thefiddler.substack.com.



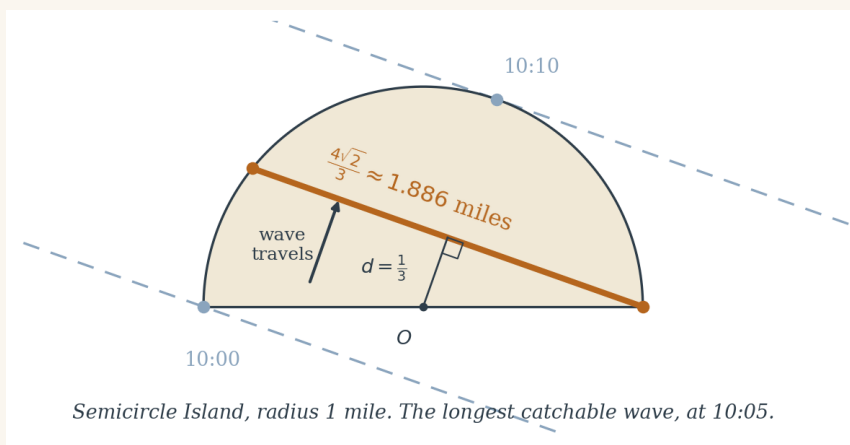
The wet sand is the stretch of the 10:05 wall lying on the island. Were the wall a proper chord, with both ends on the circle, it would span $2\sqrt{1-d^2}$, which grows as d shrinks; so we want the wall as close to O as we can bring it. The wet sand reaches that full length only while both ends stay on land, though: flatten the wave too much and the wall's lower end slides off the island into the sea, leaving a wet stretch shorter than $2\sqrt{1-d^2}$. The wall can come no closer than the moment its lower end reaches a shore corner.

At that moment the wet wall runs from the corner $(1,0)$ to the arc, both ends on the circle: a full chord. Since the corner lies on the wall, its distance from O along the travel direction, which is $\cos \alpha$, must equal d . With $d = (1 - \cos \alpha)/2$,

$$\cos \alpha = \frac{1 - \cos \alpha}{2} \implies \cos \alpha = \frac{1}{3}, \quad d = \frac{1}{3},$$

and the longest catchable wall is

$$2\sqrt{1 - \frac{1}{9}} = \frac{4\sqrt{2}}{3} \approx 1.886 \text{ miles.}$$

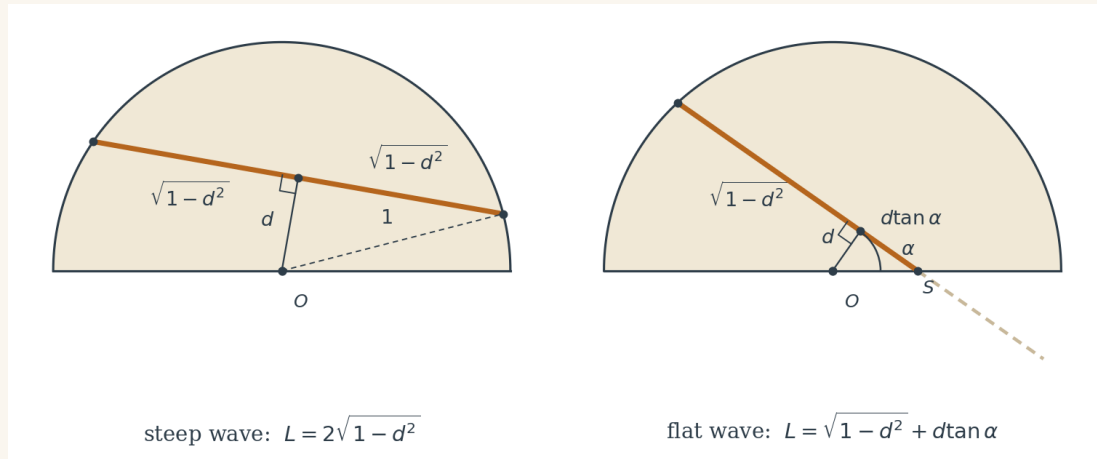


Extra credit

With a random heading we average the wet length over all directions. The island is left-right symmetric, and reversing a wave's direction leaves its 10:05 wall unmoved, so it is enough to average over α from 0 (skimming the shore) to 90° (head-on),

$$\bar{L} = \frac{2}{\pi} \int_0^{\pi/2} L(\alpha) d\alpha.$$

The wall lies a distance $d = (1 - \cos \alpha)/2$ from O , and its wet length takes one of two forms. For a steep wave the whole chord is on the island; the perpendicular from O bisects it, so each half is $\sqrt{1 - d^2}$ and $L = 2\sqrt{1 - d^2}$. For a flat wave the lower end is in the sea, and the wet part runs from the arc down to the shore: its upper piece is again $\sqrt{1 - d^2}$, and its lower piece is the side $d \tan \alpha$ of the right triangle OMS , so $L = \sqrt{1 - d^2} + d \tan \alpha$. The two regimes meet at $\alpha = \arccos \frac{1}{3}$, where the lower end sits on the corner.



To integrate, write $w = \sin \frac{\alpha}{2}$. The half-angle identity turns d into a square, which is what makes everything elementary:

$$d = \frac{1 - \cos \alpha}{2} = \sin^2 \frac{\alpha}{2} = w^2, \quad d\alpha = \frac{2 dw}{\sqrt{1 - w^2}}.$$

The square-root term then collapses, because $\sqrt{1 - d^2} = \sqrt{(1 - w^2)(1 + w^2)}$ and the $\sqrt{1 - w^2}$ cancels:

$$\begin{aligned} \int \sqrt{1 - d^2} d\alpha &= \int \sqrt{(1 - w^2)(1 + w^2)} \frac{2 dw}{\sqrt{1 - w^2}} = 2 \int \sqrt{1 + w^2} dw \\ &= w\sqrt{1 + w^2} + \ln(w + \sqrt{1 + w^2}) =: G. \end{aligned}$$

The tangent term is simpler still, since $d \tan \alpha = \frac{1}{2}(\tan \alpha - \sin \alpha)$:

$$\int d \tan \alpha d\alpha = \frac{1}{2} \int (\tan \alpha - \sin \alpha) d\alpha = \frac{1}{2}(\cos \alpha - \ln \cos \alpha) =: H.$$

Split the average at the changeover $\alpha^* = \arccos \frac{1}{3}$ and read off the two parts:

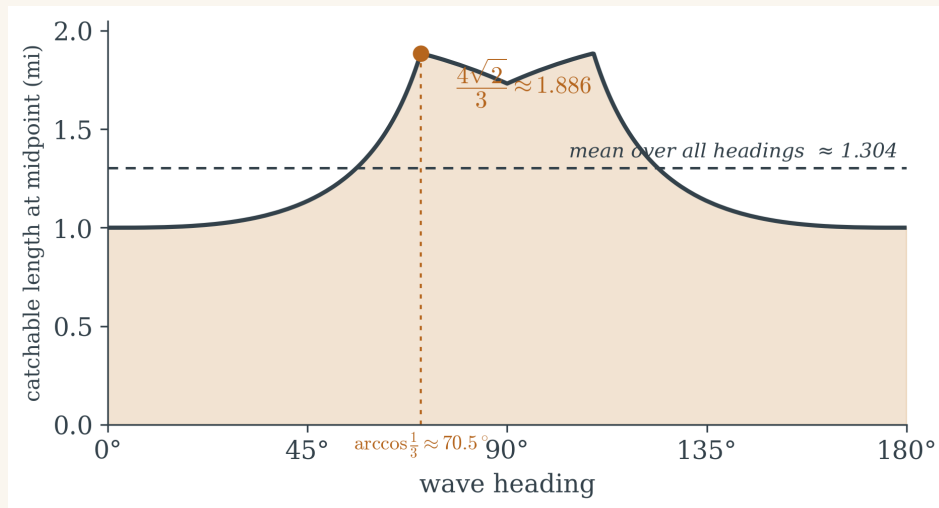
$$\frac{\pi}{2} \bar{L} = \int_0^{\alpha^*} (\sqrt{1 - d^2} + d \tan \alpha) d\alpha + \int_{\alpha^*}^{\pi/2} 2\sqrt{1 - d^2} d\alpha = [G + H]_0^{\alpha^*} + 2[G]_{\alpha^*}^{\pi/2}.$$

At the limits, with $w = 0$, $\frac{1}{\sqrt{3}}$, $\frac{1}{\sqrt{2}}$ for $\alpha = 0$, α^* , $\frac{\pi}{2}$,

$$\begin{aligned} G(0) &= 0, & H(0) &= \frac{1}{2}, \\ G(\alpha^*) &= \frac{2}{3} + \frac{1}{2} \ln 3, & H(\alpha^*) &= \frac{1}{6} + \frac{1}{2} \ln 3, \\ G\left(\frac{\pi}{2}\right) &= \frac{\sqrt{3}}{2} + \ln \frac{1+\sqrt{3}}{\sqrt{2}}. \end{aligned}$$

The flat part is $[G+H]_0^{\alpha^*} = \frac{1}{3} + \ln 3$ and the steep part is $2[G]_{\alpha^*}^{\pi/2} = \sqrt{3} - \frac{4}{3} + 2 \ln \frac{1+\sqrt{3}}{\sqrt{2}} - \ln 3$. The $\ln 3$ cancels and $2 \ln \frac{1+\sqrt{3}}{\sqrt{2}} = \ln(2 + \sqrt{3})$, so $\frac{\pi}{2} \bar{L} = \sqrt{3} - 1 + \ln(2 + \sqrt{3})$ and

$$\bar{L} = \frac{2}{\pi} (\sqrt{3} - 1 + \ln(2 + \sqrt{3})) \approx \boxed{1.304 \text{ miles.}}$$



The curve has two peaks of $\frac{4\sqrt{2}}{3}$ at the best heading on each side, a dip to $\sqrt{3}$ for the head-on wave, and a floor of 1 mile, the radius, for a wave skimming the shore.

Python code

The code follows the puzzle directly: for each heading it builds the 10:05 wall and measures the part lying on the island. The longest over all headings is the main answer; the average over all headings is the extra credit, and it matches the closed form.

```
import numpy as np
from math import sqrt, log, pi

def wet_length(alpha, n=400001):
    u = np.array([np.cos(alpha), np.sin(alpha)]) # travel direction
    t = np.array([-np.sin(alpha), np.cos(alpha)]) # along the wavefront
    reach = lambda v: 1.0 if v[1] >= 0 else abs(v[0]) # support of the half-disk
    c = 0.5 * (reach(u) - reach(-u)) # 10:05 wall: distance c from 0
    s = np.linspace(-2, 2, n) # march along the wall
    p = c * u + np.outer(s, t)
    land = (p[:, 0]**2 + p[:, 1]**2 <= 1) & (p[:, 1] >= 0) # on the island?
```

```
    return 0.0 if not land.any() else s[land].max() - s[land].min()

grid = np.linspace(0, 2 * pi, 7201)
lengths = np.array([wet_length(a) for a in grid])

closed = (2 / pi) * (sqrt(3) - 1 + log(2 + sqrt(3)))
print(f"longest: {lengths.max():.4f} (4*sqrt(2)/3 = {4 * sqrt(2) / 3:.4f}")
print(f"average: {lengths.mean():.4f} (closed form = {closed:.4f}")
# longest: 1.8855 (4*sqrt(2)/3 = 1.8856)
# average: 1.3044 (closed form = 1.3044)
```