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Can You Cheat on the Quiz?

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A puzzle from Fiddler on the Proof, in which a rule against repeats leaks information two questions away.

Problem

Charlie faces a pop quiz of three multiple-choice questions, each with four options A to D and worth one point. He has no idea of any answer, but he knows one thing about his teacher: consecutive questions never share a correct answer. He glimpses his neighbour's answer to the first question before the neighbour covers the page. On average, what score can he expect?

We take every answer key that obeys the teacher's rule to be equally likely. That is the same as choosing the first answer at random and then each later answer at random from the three that differ from the one before it.

Solution

The first question is worth a sure point: Charlie copies it.

The second question cannot repeat the first, so its answer is one of the other three, each equally likely. Any of those three is a one-in-three guess.

The third question is where the rule starts to pay, even though the second answer is unknown. Suppose the first answer was A. The second is then B, C or D, and whichever it is, the third can be A. So A is never ruled out at the third question, while each of B, C and D is ruled out whenever it happens to be the second answer, which is a third of the time:

$$\Pr(\text{third} = \text{A}) = \tfrac{1}{3}, \qquad \Pr(\text{third} = \text{B}) = \tfrac{2}{3} \cdot \tfrac{1}{3} = \tfrac{2}{9},$$

and likewise for C and D. The letter Charlie saw is the best guess for the third question, and it is right a third of the time.

Expected score adds up question by question, so the best plan is simply the best guess for each question separately:

$$\boxed{\mathbb{E}[\text{score}] \;=\; 1 + \tfrac{1}{3} + \tfrac{1}{3} \;=\; \tfrac{5}{3}.}$$

Extra Credit

Now the quiz has seven questions, with the same rule, and Charlie may look at exactly one answer. Which should he look at, and what score should he then expect?

How the rule echoes along the quiz

Call the answer Charlie sees a, and ask how likely a question d places away is to share it. Write p_d for that chance. One step on, a question matches a only if the one before it did not, and then with chance $\frac{1}{3}$, so

$$p_{d+1} \;=\; \tfrac{1}{3}\,(1-p_d), \qquad p_0 = 1.$$

Its fixed point is $\frac{1}{4}$, and the gap from it shrinks by a factor of $-\frac{1}{3}$ each step, so

$$p_d \;=\; \tfrac{1}{4} + \tfrac{3}{4}\left(-\tfrac{1}{3}\right)^d.$$

The sign alternates. At even distances the seen letter is more likely than a quarter, and it is Charlie's best guess, right with chance $\frac{1}{4} + \frac{3}{4} \cdot 3^{-d}$. At odd distances it is less likely than a quarter, and the best guess is any other letter, right with chance $\frac{1}{4} + \frac{1}{4} \cdot 3^{-d}$.

Now put $d = 2j-1$ and $d = 2j$ side by side:

$$\tfrac{1}{4} + \tfrac{1}{4} \cdot 3^{-(2j-1)} \;=\; \tfrac{1}{4} + \tfrac{3}{4} \cdot 3^{-2j}.$$

The best chance at an odd distance equals the best chance one step further on. The information decays in pairs: $\frac{1}{3}, \frac{1}{3}$ at distances one and two, then $\frac{7}{27}, \frac{7}{27}$, then $\frac{61}{243}, \frac{61}{243}$.

Where to look

A glimpsed answer helps the questions near it most, so the middle of the quiz is the natural place to look. Looking at question four leaves distances $1, 1, 2, 2, 3, 3$ for the other six questions. But because distances pair up, looking at question three, with distances $2, 1, 1, 2, 3, 4$, trades a distance of three for a distance of four and loses nothing. Question five is the mirror image. So three positions tie, and all three give

$$\boxed{\mathbb{E}[\text{score}] \;=\; 1 + 4 \cdot \tfrac{1}{3} + 2 \cdot \tfrac{7}{27} \;=\; \tfrac{77}{27} \;\approx\; 2.852,}$$

for question three, four or five. Question two gives $\frac{673}{243}$ and question one only $\frac{653}{243}$, because a glimpse at the end of the quiz wastes half its reach on questions that do not exist.

The puzzle's wording invites a single answer, and the middle question is the one most solvers will give. It is a correct answer. It is just not the only one.

The computation

With four letters the answer keys can simply be listed: 36 keys obey the rule for three questions and 2916 for seven. For each question Charlie might look at, and each letter he might see, the check tallies how often every other question takes each letter and takes the most common one as his guess, all in exact fractions.

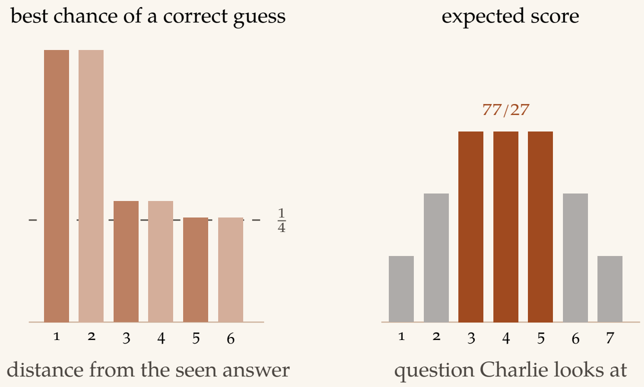

Figure 1: *Left: Charlie's best chance at a question d places from the answer he saw, falling in equal pairs towards $\frac{1}{4}$. Right: his expected score by which of the seven answers he looks at. The top is flat across questions three, four and five.*

```python
from fractions import Fraction
from itertools import product
from collections import Counter

def keys(n):                # every key with no consecutive repeat
    return [s for s in product(range(4), repeat=n)
            if all(s[i] != s[i+1] for i in range(n-1))]

def expected_score(n, k):   # Charlie looks at question k (0-based)
    K, total = keys(n), Fraction(0)
    for a in range(4):
        cond = [s for s in K if s[k] == a]
        score = Fraction(1)
        for j in range(n):
            if j != k:
                c = Counter(s[j] for s in cond)
                score += Fraction(max(c.values()), len(cond))
        total += Fraction(len(cond), len(K)) * score
    return total

print(expected_score(3, 0))                # 5/3
print([expected_score(7, k) for k in range(7)])
# [653/243, 673/243, 77/27, 77/27, 77/27, 673/243, 653/243]
```

The enumeration returns 5/3 for the three-question quiz and confirms the three-way tie at 77/27, independently of the recurrence that predicted it.