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# *Can You Win at Asymmetric Bingo?*

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A puzzle from Fiddler on the Proof, in which the smaller board is the better one.

## *Problem*

You play bingo on a $5 \times 5$ board while your opponent plays on an $8 \times 8$ board. Their board carries the numbers 1 through 64 in a random arrangement. Yours carries 25 numbers chosen at random, without replacement, from the same range, also arranged at random. There are no free squares.

Numbers are drawn one at a time from 1 to 64, without replacement. If the drawn number is on your board you mark it; otherwise your opponent, who is certain to have it, marks it on theirs. The game ends when somebody has five markers in a row, going across, down, or diagonally.

Who is more likely to win? And with what probability do you win?

## *Your board is also a mask on theirs*

The setup looks lopsided in your opponent's favour. They have every number, so they mark roughly 39 of the 64 draws to your 25. They have 96 places to make five in a row against your 12. On both counts they are ahead.

Both counts mislead, and the second one is where the puzzle lives.

Your opponent holds all 64 numbers, but 25 of them are on your board, and those you take first. The squares carrying them are dead: your opponent will never mark them, not once in the whole game. So their board is not an $8 \times 8$ of usable squares. It is an $8 \times 8$ with 25 holes punched through it at random, and five in a row is a brittle thing to ask of a holed board.

Count the windows. On the $8 \times 8$ there are $8 \times 4 = 32$ horizontal runs of five, another 32 vertical, and 32 diagonal, since each diagonal direction offers a $4 \times 4$ grid of starting squares. That is 96 in all. A particular window survives only if all five of its squares are live, and the 25 dead squares are a random 25-subset of the 64, so

$$\Pr(\text{a given window survives}) = \frac{\binom{39}{5}}{\binom{64}{5}} = \frac{575757}{7624512} = 0.0755\ldots$$

By linearity the expected number of surviving windows is

$$96 \times \frac{\binom{39}{5}}{\binom{64}{5}} = 7.25.$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 21 | 36 | 24 | 2 | 20 | 23 | 47 | 17 |
| 53 | 10 | 31 | 57 | 62 | 52 | 19 | 4 |
| 64 | 22 | 7 | 54 | 28 | 27 | 37 | 1 |
| 60 | 42 | 30 | 59 | 55 | 32 | 34 | 46 |
| 44 | 35 | 63 | 8 | 45 | 3 | 41 | 15 |
| 58 | 13 | 16 | 14 | 43 | 9 | 49 | 38 |
| 5 | 18 | 6 | 39 | 40 | 48 | 56 | 50 |
| 29 | 51 | 11 | 33 | 26 | 25 | 61 | 12 |

Shaded squares carry the 25 numbers on my board, so the opponent can never mark them. Only 7 of the 96 five-in-a-row windows survive here; two are outlined.

Ninety-six windows collapse to about seven. Your twelve lines, meanwhile, are never holed at all, because every square of your board is one you will eventually mark. So the true comparison is not 96 against 12. It is about 7 against 12, and you are ahead.

## *Why the extra draws do not help*

The other apparent advantage, that your opponent marks more squares, turns out to be worth nothing, and it is worth seeing exactly why.

Fix any five numbers. Whoever needs them, that line is completed at the moment the last of those five is drawn, and the five draw positions are five distinct places in a uniformly random ordering of 64. That law does not care whose board the numbers sit on. A window needing five numbers is finished at the same moment in distribution whether it belongs to you or to them.

So marking more squares does not make your opponent faster. It only means their markers are scattered over more squares, most of which are not helping any surviving window. Both players are waiting for the same kind of event, the arrival of the last of a particular five, and the only thing that differs is how many such fives each of you is waiting on. That is the whole asymmetry, and it runs the other way from the board sizes.

$$\boxed{\text{You, on the } 5 \times 5, \text{ are more likely to win.}}$$

## *The computation*

Two structural claims carry the argument, and both were checked rather than asserted.

The count of surviving windows comes out at 7.2627 in simulation against the exact $96\binom{39}{5}/\binom{64}{5} = 7.2493$. Its spread is wide, standard deviation 3.25 about that mean, and in about one game in 240 no window survives at all, so your opponent cannot win however the draws fall.

The race itself was simulated by fixing the draw order to $1, 2, \ldots, 64$, which the labels allow, and randomising only the boards.

```
import numpy as np

mine = np.argsort(rng.random((M, 64)), axis=1)[:, :25] # my cells -> numbers
opp  = np.argsort(rng.random((M, 64)), axis=1)    # their cells -> numbers

my_time = np.full(M, BIG, np.int16)               # first of my 12 lines
for ln in L5:
    np.minimum(my_time, mine[:, ln].max(axis=1), out=my_time)

on_mine = np.zeros((M, 64), bool)
np.put_along_axis(on_mine, mine, True, axis=1)
live = ~np.take_along_axis(on_mine, opp, axis=1) # their markable squares

opp_time = np.full(M, BIG, np.int16)              # first surviving window
for ln in L8:
    ok = live[:, ln].all(axis=1)
    np.minimum(opp_time, np.where(ok, opp[:, ln].max(axis=1), BIG), out=opp_time)

wins = (my_time < opp_time).sum()
```

You win a clear majority, which settles the question asked. Left unopposed you would finish on the 38th draw at the median and your opponent on the 43rd, while the game itself ends on the 35th on average. The same game was also simulated the slow way, one draw at a time with every line tested after every draw, and twenty thousand such games agree well within their own noise.

One wrong version is worth recording for how it announced itself. An early draft built my board from the first twenty-five numbers *drawn* rather than from an independent random subset, handing me all my numbers in the opening twenty-five draws and winning every game at a median finishing draw of five. A board needing five distinct numbers cannot be completed on the fifth draw, and that impossibility is what exposed the bug.

## *Extra Credit*

*To at least the nearest hundredth, what is the probability that you will win a given game of asymmetric bingo?*

### *Solution*

There is no clean closed form here, and it is worth saying why rather than just reporting a number.

Each side finishes at a minimum, over its viable lines, of the last arrival among five numbers. That would be tractable if the lines were independent, and on neither side are they. Your twelve lines share squares with one another, every cell of the $5 \times 5$ lying on two of them or three. The surviving windows on the $8{\times}8$ overlap far more heavily, since consecutive windows along a row share four squares out of five. And which windows survive is itself random and correlated with everything else, depending as it does on the same twenty-five numbers that make up your own lines. Nothing factorises, so the value comes from playing the game out.

Thirty-five million games give

$$\boxed{\Pr(\text{you win}) \;=\; 0.6657 \;\approx\; 0.67.}$$

*Rounding, and why it needs saying*

The question asks for the nearest hundredth, and the answer lands close enough to a boundary that the third decimal decides the second, so it deserves an error bar. Two runs from independent seeds gave $0.665568 \pm 0.000149$ over ten million games and $0.665684 \pm 0.000094$ over twenty-five million, pooling to

$$0.665651 \pm 0.000080,$$

about eight standard errors clear of 0.665. The hundredth really is 0.67, but only just: ten million games alone put it under four standard errors from the boundary, which is not enough to name a hundredth with confidence, and a shorter run could report 0.66 in perfectly good faith.